Lecture 3: Message Authentication Codes

Last time: CPA secure encryption

Today:

1. Recap
2. Motivate and define the notion of message authentication codes (MACs)
3. Construct MACs

Recap:

Definition (informal): A symmetric encryption scheme (Enc, Dec) Is said to be secure against adaptive chosen message attack if for any PPT adv A, any messages $m_1, \dots, m_t \in M$ and any $m'_1, \dots, m'_t \in M$ chosen adaptively by A

$$Enc(k, m_1), \dots, Enc(k, m_t) \approx Enc(k, m'_1), \dots, Enc(k, m'_t)$$

Construction: Using PRF and one-time pad.

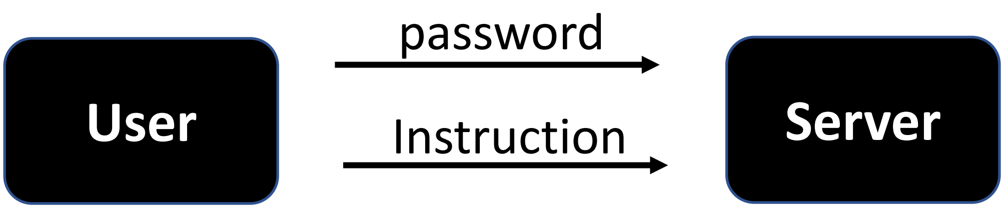
In practice: Using AES and one-time pad with counter mode:

$$Enc(k, m_1 || \dots || m_n) = r, \{AES(k, r+i) \oplus m_i\}_{i \in [n]}$$

This definition does not provide any form of authentication!

Namely, an adversary may chance the message and the parties may not be able to detect it. This is a security breach!

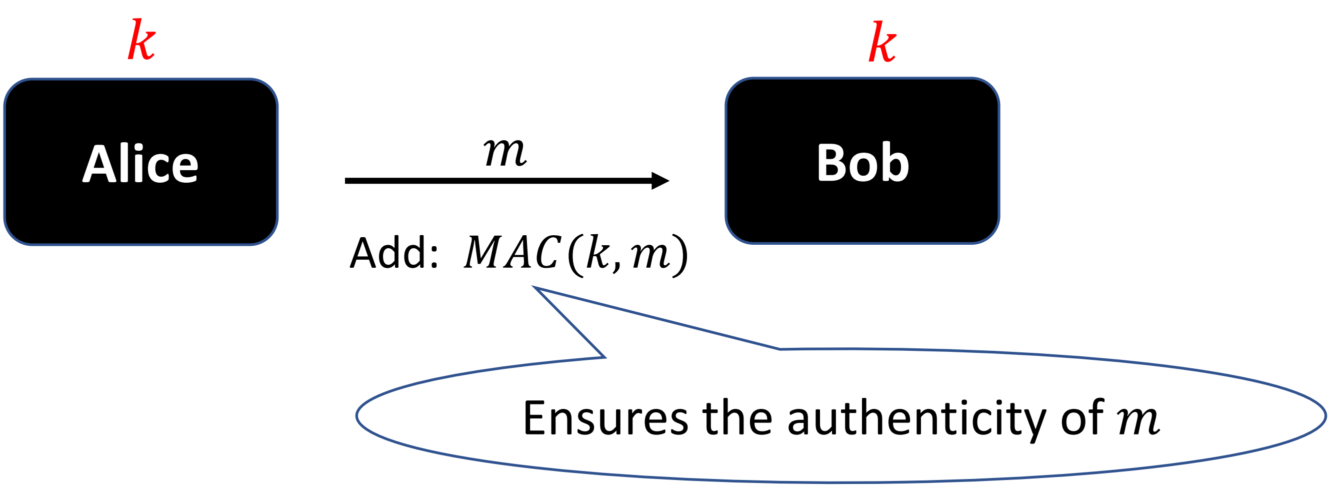
Authentication:



How does the server know that it is Alice who is sending the instruction?

Message Authentication Codes (MACs)

Assumes the communicating parties share a secret key k.



Attacker goal: Existential forgery; i.e., forge a MAC for any message.

Is this goal too strong? Why do we care if the attacker MACs gibberish?

Parties can MAC their secret key (which is gibberish)

Attacker power: See MACs for messages of its choice.

Definition: A message authentication code consists of a (signing) function $MAC: K \times M \to \{0,1\}^n$ with the following security guarantee: For any PPT adversary A, it wins in the following game with only negligible probability.

Challenger $\qquad\qquad$ $\boldsymbol{A}$

$$\xleftarrow{\quad m_1 \quad}$$

$$\xrightarrow{\quad MAC(k, m_1) \quad}$$

$$\xleftarrow{\quad m_2 \quad}$$

$$\xrightarrow{\quad MAC(k, m_2) \quad}$$

$$\vdots$$

$$\xleftarrow{\quad m_k \quad}$$

$$\xrightarrow{\quad MAC(k, m_k) \quad}$$

$$\xleftarrow{\quad m^*, t^* \quad}$$

A wins iff $t^* = MAC(k, m^*)$ and $m^* \notin \{m_1, \dots, m_k\}$

Definition: A $MAC: K \times M \to \{0,1\}^n$ is secure against adaptive chosen message attacks if any PPT adversary wins in the above game with only negligible probability.

Remark: More generally, one can define a MAC as two algorithms: a signing algorithm $Sig: K \times M \to \{0,1\}^n$ and a verification algorithm $Ver: K \times M \times \{0,1\}^n \to \{0,1\}$, such that for every $k \in K$ and $m \in M$,

$$Ver\big(k, m, Sig(k, m)\big) = 1$$

We chose to define a single algorithm (MAC) since that is the case in practice. In the public key setting we will define it via two algorithms as above (stay tuned!).

Constructions: Use a PRF!

$$\boldsymbol{MAC(k, m) = PRF(m)}$$

Note: The value of a PRF is required to look **random**, whereas the value of a MAC is required to be **unpredictable** (both given oracle access to the function). The latter is weaker if the output size is large.

Theorem: Every PRF with domain D and range R, such that $1/|R|$ is negligible is a MAC with message space D.

Corollary: AES is a secure MAC for messages in $\{0,1\}^{128}$.

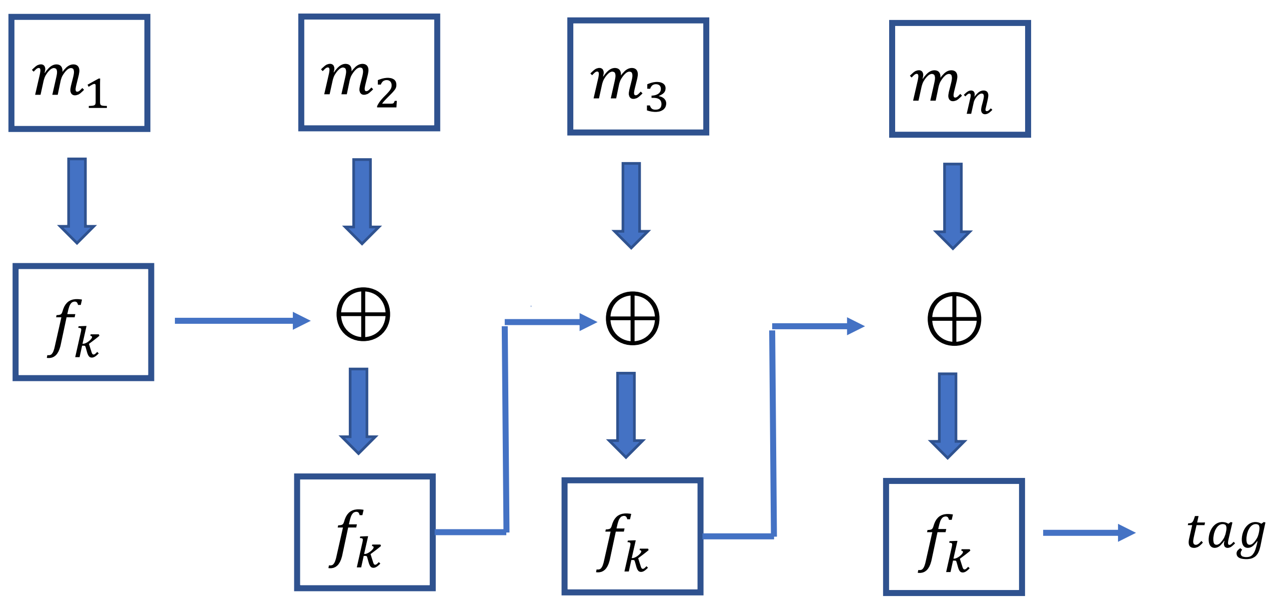
Authenticating arbitrarily long messages:

Attempt 1: Partition the message into smaller blocks and MAC each block separately.

$$\boldsymbol{m = (m_1, \dots, m_n) \in D^n} \Longrightarrow \boldsymbol{MAC(k, m_1), \dots, MAC(k, m_n)}$$

Insecure! Can execute a mix and match attack.

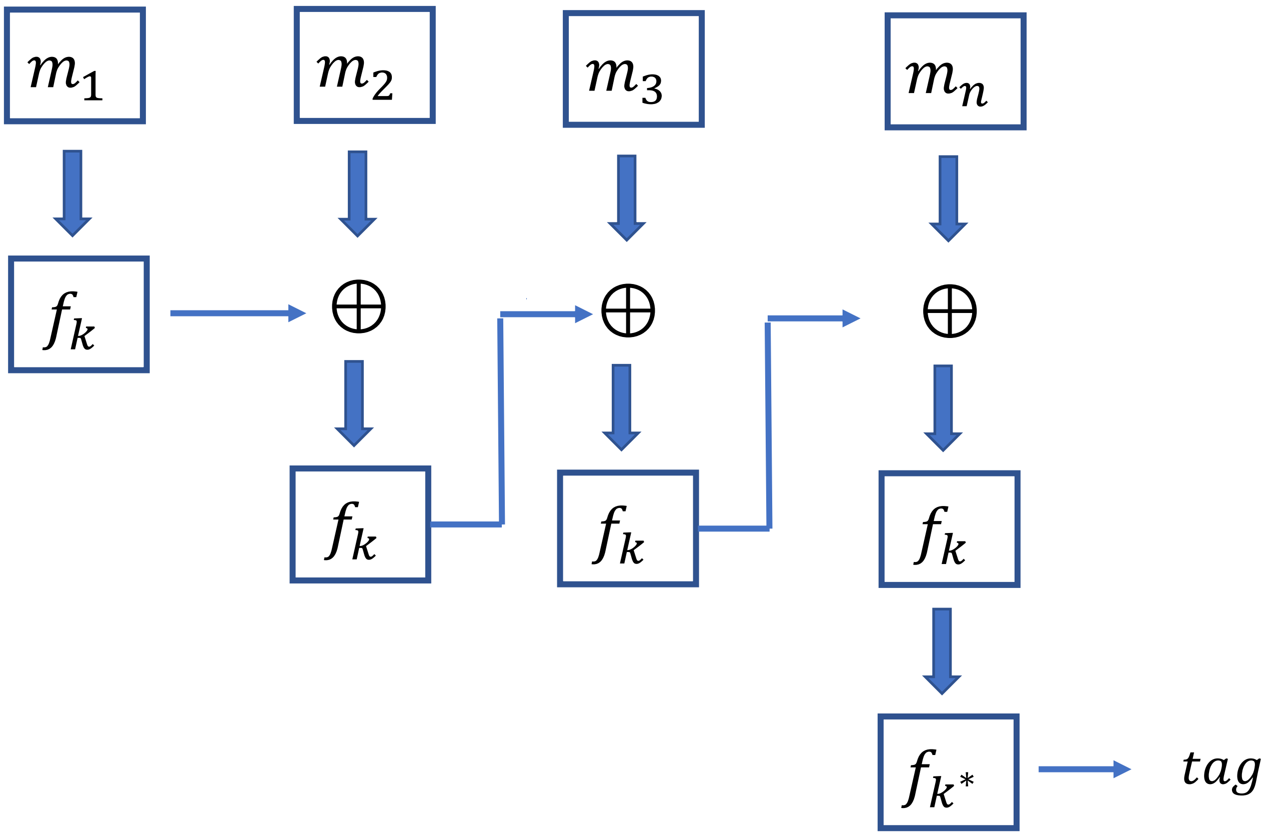
Attempt 2: Partition the message into smaller blocks and MAC each block using a chaining:



Insecure! Can execute an extension attack.

Given a tag for m, denoted by tag, and given a tag for m', denoted by tag', one can generate a tag for $m'||tag' \oplus m$. The tag is tag.

Final fix: Choose two independent and random keys $k, k^* \in K$.



This MAC is known as **Cipher Block Changing (CBC) MAC**, and secure against adaptive chosen message attack if f is a PRF.

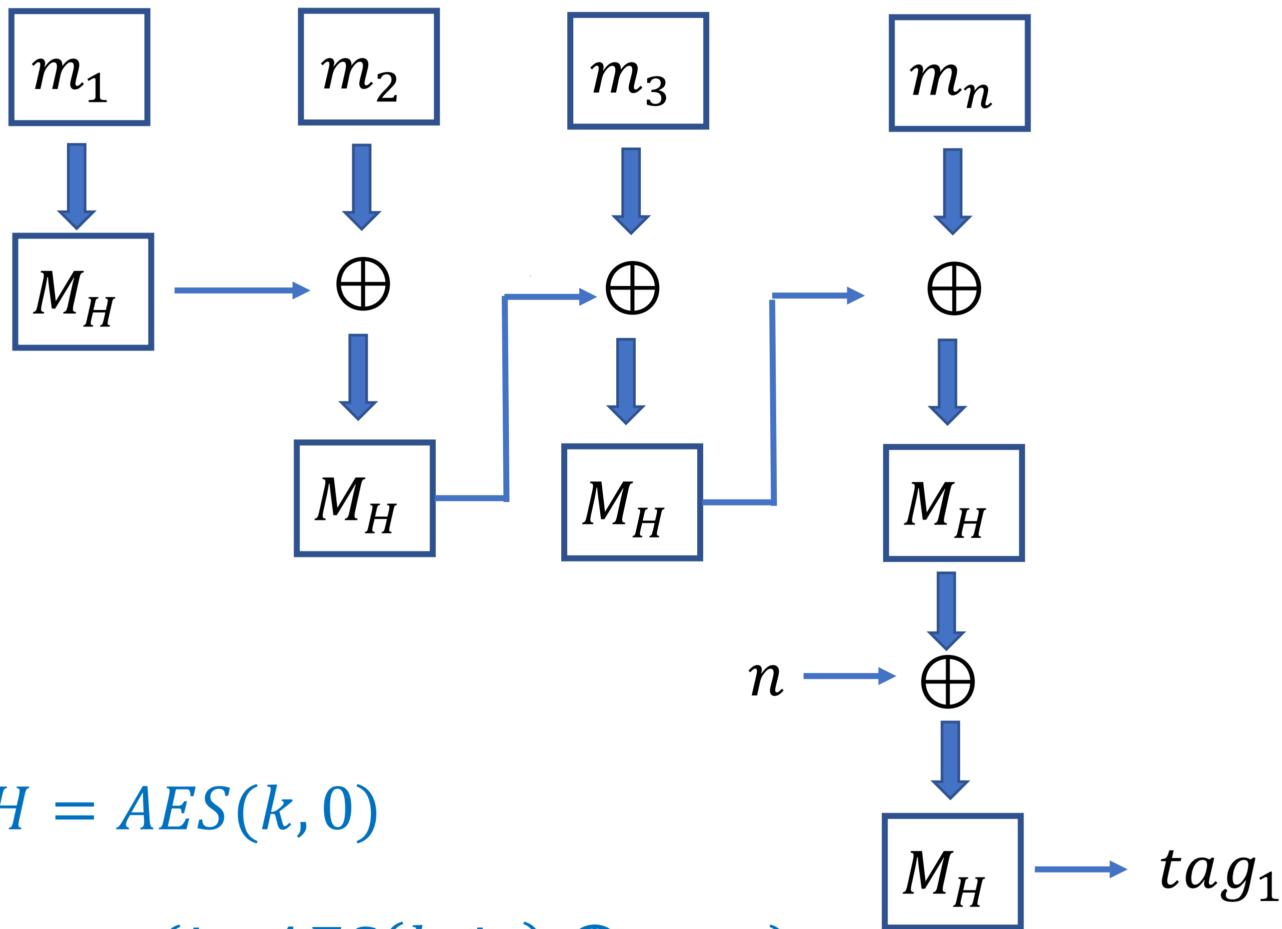
MAC with improved efficiency: Galois MAC (GMAC)

Basic idea: Use the same chaining structure as above, but instead of using a PRF (i.e., AES), use a one-time secure MAC, and encrypt the tag using AES.

Namely: Instead of using AES, use the multiplication function

$$M_H\colon \{0,1\}^{128} \to \{0,1\}^{128}$$

defined by $M_H(x) = H \cdot x$ where $H \in \{0,1\}^{128}$ and multiplication is in the Galois field $GF[2^{128}]$



$$H = AES(k, 0)$$

$$tag = (iv, AES(k, iv) \oplus tag_1)$$

Note that $MAC(k, m)$ is computed as follows:

1. Parse $m = m_1 || \dots m_n$.
2. Compute $\mathrm{v} = \sum_i m_i H^i$ over $GF[2^{128}]$
3. Output an encryption of $H \cdot (v \oplus n)$

Efficiency gain: H^i can be precomputed, and multiplication is more efficient that AES.